Chapter 11
Circumference, Area, and Volume

11.1 Circumference and Arc Length

11.2 Areas of Circles and Sectors

11.3 Areas of Polygons

11.4 Three-Dimensional Figures

11.5 Volumes of Prisms and Cylinders

11.6 Volumes of Pyramids

11.7 Surface Areas and Volumes of Cones

11.8 Surface Areas and Volumes of Spheres

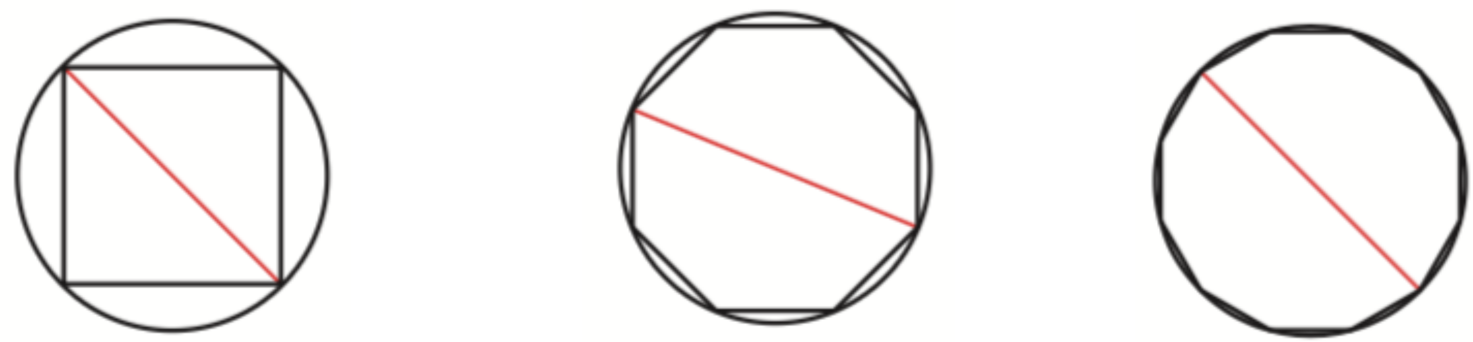
11.1 Circumference and Arc Length

Circumference

The distance around the circle.

Consider a regular polygon inscribed inside a circle.

Increase the number of sides of the polygon.



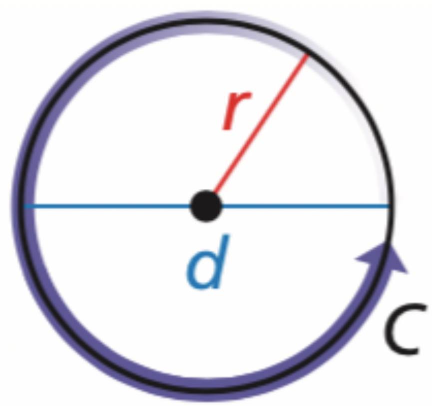
The ratio of perimeter (C) over diameter (d) approaches π.

$$\frac{C}{d} = 3.1415926...$$

11.1 Circumference and Arc Length

Circumference

The circumference is the diameter times π.



$$C = \pi d = 2\pi r$$

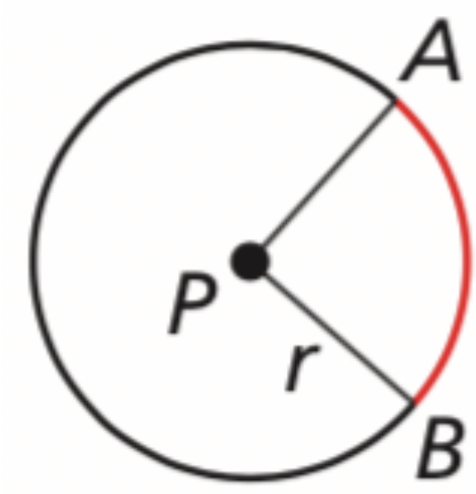
More commonly, it is two times radius times π.

11.1 Circumference and Arc Length

Arc Length

The arc length is a portion of the circumference of a circle. Use the arc (in degrees) to find its length.

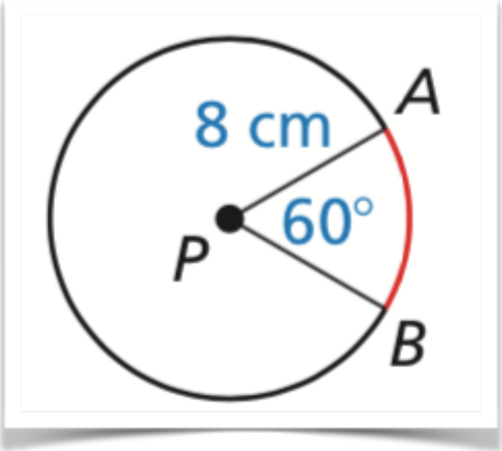
$$\text{Arc length} = 2\pi r\left(\frac{m\overset{\frown}{AB}}{360^\circ}\right)$$



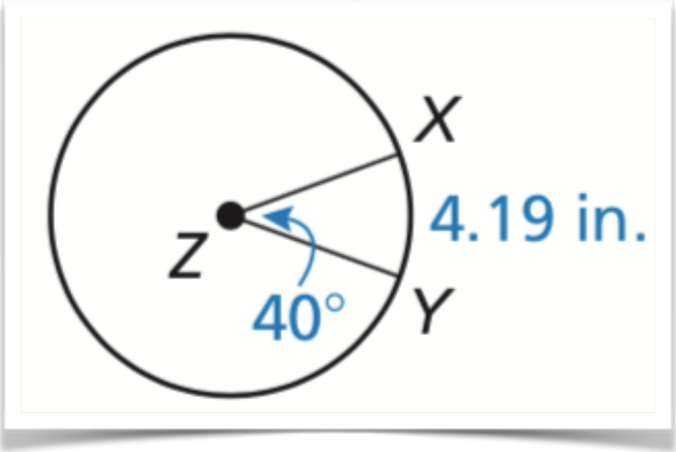
11.1 Circumference and Arc Length

Calculate values

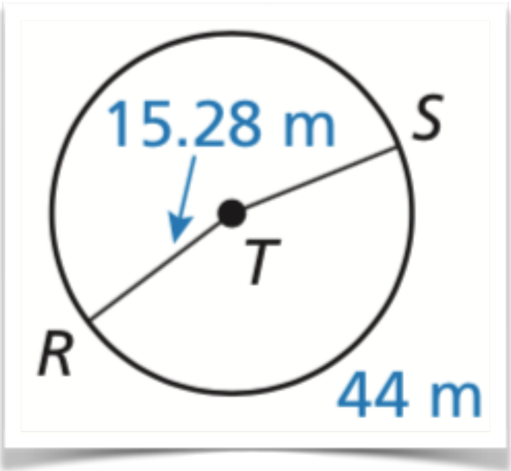
a) Arc length of $m\overarc{AB}$



b) Circumference of $\odot Z$



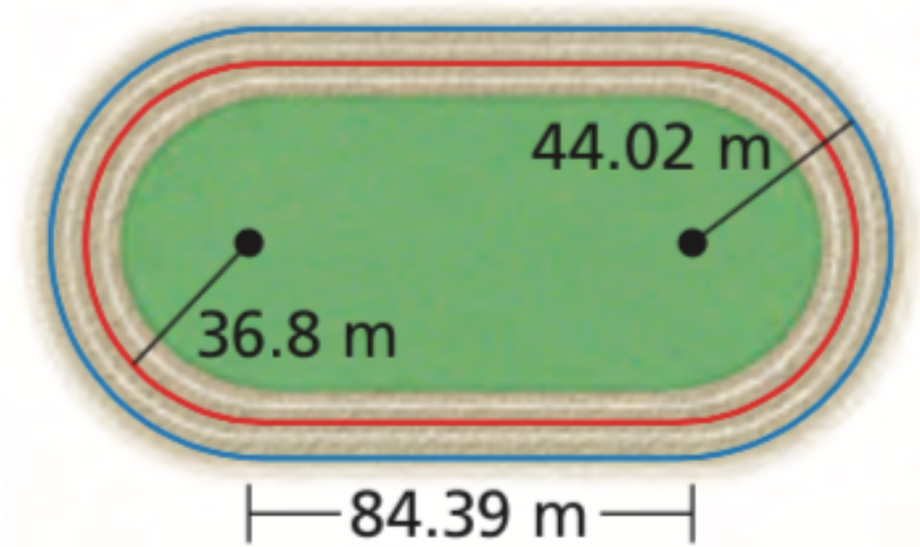
c) Arc angle $m\overarc{RS}$



11.1 Circumference and Arc Length

Real World

A runner runs around the track below. The ends of the track are semicircle arcs with each radius listed. How far is one lap around the track?



11.1 Circumference and Arc Length

Using Circumference

The circumference of a circle is $2\pi r$

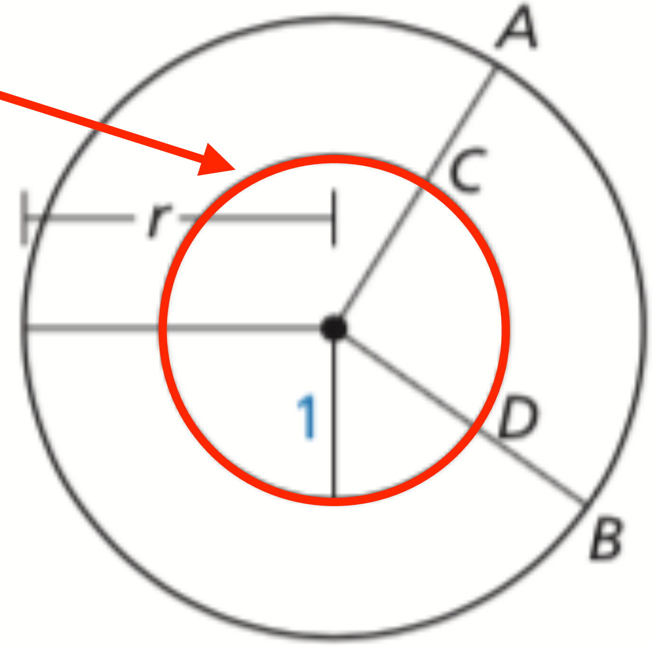
If the radius is equal to 1 (smaller red circle), then the circumference is 2π

This is called the **unit circle** (r = 1).

The arc length of $\overset{\frown}{CD}$

$$= circumference\left(\frac{m\overset{\frown}{CD}}{360^\circ}\right)$$

$$= 2\pi\left(\frac{m\overset{\frown}{CD}}{360^\circ}\right)$$



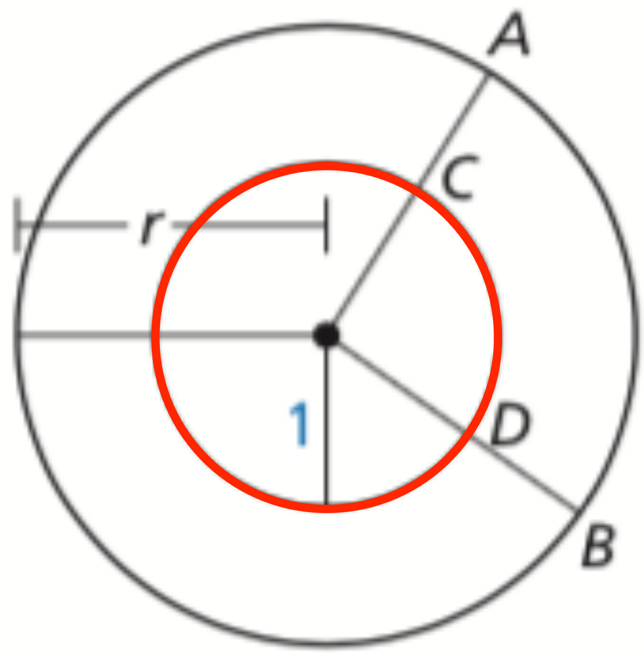
The arc length of $\overset{\frown}{CD} = 2\pi\left(\frac{m\overset{\frown}{CD}}{360^\circ}\right)$

11.1 Circumference and Arc Length

Using Circumference

All circles are similar, so their dimensions are proportional.

$$\frac{\text{arc length of } \widehat{AB}}{\text{arc length of } \widehat{CD}} = \frac{r}{1}$$



$$\text{arc length of } \widehat{AB} = r \cdot \text{arc length of } \widehat{CD}$$

$$\text{arc length of } \widehat{AB} = r \cdot 2\pi\left(\frac{m\widehat{CD}}{360^\circ}\right)$$

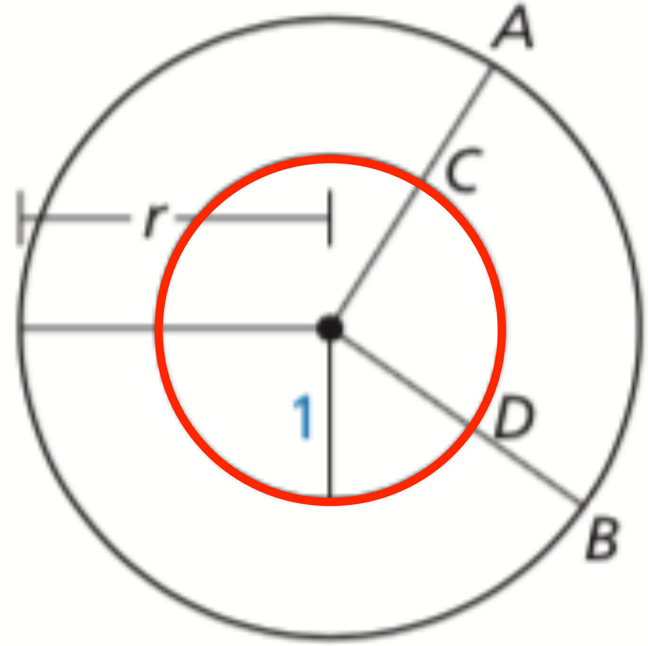
The arc length of $\widehat{CD} = 2\pi\left(\frac{m\widehat{CD}}{360^\circ}\right)$

11.1 Circumference and Arc Length

Radians

- Radians is the length around the *unit circle* (r = 1).
- 2π radians is the circumference of a *unit circle*.
- Therefore, equivalent units are

$360^\circ = 2\pi$ radians or $\frac{2\pi}{360^\circ}$



11.1 Circumference and Arc Length

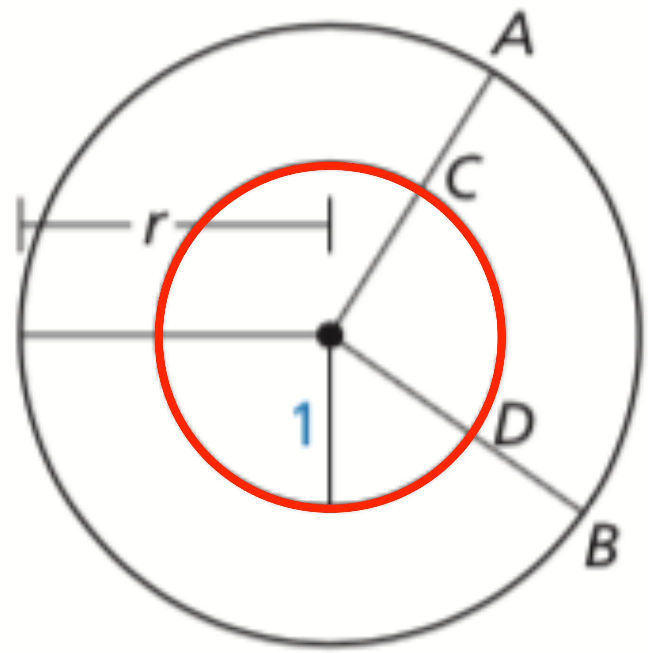
Radians

- Converting from degrees to radians

$$\text{degrees}\left(\frac{2\pi}{360^\circ}\right) = \text{radians}$$

- Converting from radians to degrees

$$\text{radians}\left(\frac{360^\circ}{2\pi}\right) = \text{degrees}$$



11.1 Circumference and Arc Length

Examples

$$\text{degrees}\left(\frac{2\pi}{360^\circ}\right) = \text{radians}$$

$$\text{radians}\left(\frac{360^\circ}{2\pi}\right) = \text{degrees}$$

a) Convert 45° to radians

b) Convert $\frac{3\pi}{2}$ radians to degrees

c) Calculate the arc length of an arc on a circle with radius of 5 meters and arc angle of $\frac{2\pi}{3}$ radians.